

# **FÍSICA GERAL E EXPERIMENTAL I**

RESOLUÇÃO DA LISTA IV

**UNIVERSIDADE CATÓLICA DE GOIÁS**

Departamento de Matemática e Física

Disciplina: Física Geral e Experimental I (MAF 2201)

RESOLUÇÃO DA LISTA IV

1.

$$a) x_{cm} = \frac{x_1 m_1 + x_2 m_2 + x_3 m_3}{m_1 + m_2 + m_3} = \frac{0.3 + 1.8 + 2.4}{3 + 8 + 4} \Rightarrow x_{cm} = 1,1m$$

$$b) y_{cm} = \frac{y_1 m_1 + y_2 m_2 + y_3 m_3}{m_1 + m_2 + m_3} = \frac{0.3 + 2.8 + 1.4}{3 + 8 + 4} = 1,3m$$

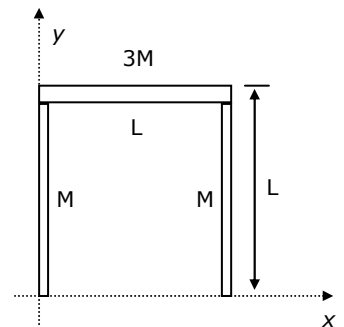
c) ele se desloca em direção a essa partícula

2.

Podemos considerar as hastes como partícula colocadas no centro de massa de cada uma delas

$$a) x_{cm} = \frac{x_1 m_1 + x_2 m_2 + x_3 m_3}{m_1 + m_2 + m_3} \Rightarrow \frac{0.M + \frac{L}{2}.3M + L.M}{M + 3M + M} = \frac{L}{2}$$

$$b) y_{cm} = \frac{y_1 m_1 + y_2 m_2 + y_3 m_3}{m_1 + m_2 + m_3} \Rightarrow \frac{\frac{L}{2}.M + L.3M + \frac{L}{2}.M}{5M} = \frac{4LM}{5M} = 0,8L$$



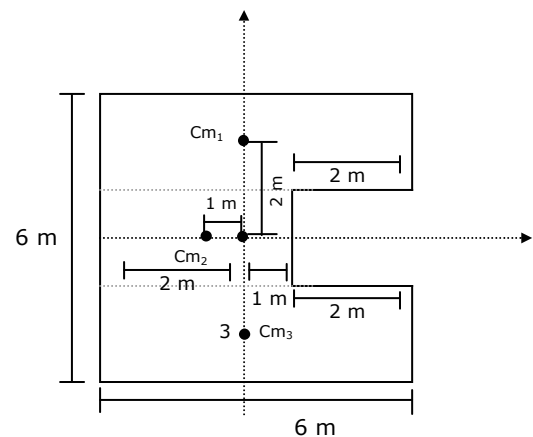
3.

Dividindo a placa em 3 partes, podemos considerar cada parte como partícula colocada em seu centro de massa

$$x_{cm} = \frac{x_1 m_1 + x_2 m_2 + x_3 m_3}{m_1 + m_2 + m_3} \Rightarrow \frac{0.m_1 + 1.m_2 + 0.m_3}{m_1 + m_2 + m_3}$$

 temos que:  $m_1 = m_3 = m$  e  $m_2 = \frac{2m}{3}$ 

$$\Rightarrow x_{cm} = \frac{-1 \cdot \frac{2m}{3}}{m + \frac{2m}{3} + m} = \frac{-\frac{2m}{3}}{\frac{8m}{3}} = \frac{-2m}{3} \cdot \frac{3}{8m} = \frac{-2}{8} = -0,25m$$



$$y_{cm} = \frac{y_1 m_1 + y_2 m_2 + y_3 m_3}{m_1 + m_2 + m_3} \Rightarrow \frac{2 \cdot m_1 + 0 \cdot m_2 + -2 \cdot m_3}{m_1 + m_2 + m_3} = \frac{2m - 2m}{m + \frac{2m}{3} + m}$$

$$\Rightarrow y_{cm} = 0$$

4.

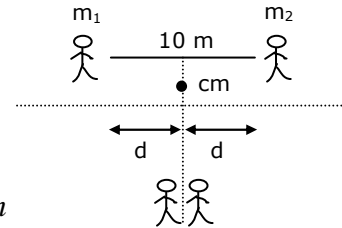
$$m_1 = 65 \text{ kg}, \quad m_2 = 40 \text{ kg}$$

$$\sum F_{ex} = 0 \Rightarrow c_m \Rightarrow \text{está em repouso}$$

$$\dot{x}_{cm} = x_{cm}$$

$$\Rightarrow \frac{x_1 m_1 + x_2 m_2}{m_1 + m_2} = \frac{d(m_1 + m_2)}{m_1 + m_2} \Rightarrow d = \frac{0.65 + 10 \cdot 60}{65 + 40} = 3,81 \text{ m}$$

$$d' = 10 - 3,81 \Rightarrow d' = 6,19 \text{ m}$$



5.

$$m_c = 2400 \text{ kg}, \quad v_c = 80 \text{ km/h}, \quad m_f = 1600 \text{ kg}, \quad v_f = 60 \text{ km/h}, \quad v_{cm} = ?$$

$$M \cdot \vec{V}_{cm} = m_c \cdot \vec{v}_c + m_f \cdot \vec{v}_f, \text{ como as velocidades estão na mesma direção, temos que:}$$

$$\Rightarrow (2400 + 1600) \cdot v_{cm} = 2400 \cdot 80 + 1600 \cdot 60 \Rightarrow v_{cm} = 72 \text{ km/h}$$

6.

$$v_{01} = v_{02} = 0, \quad y_{01} = y_{02} = 0, \quad t_1 = 300 \text{ ms} = 0,3 \text{ s} \quad e \quad t_2 = 200 \text{ ms} = 0,2 \text{ s}$$

as pedras estão em queda livre

a)

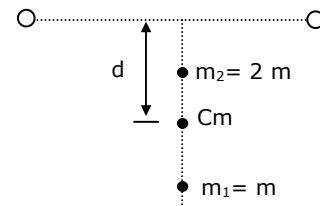
$$y = y_0 + v_0 t - \frac{1}{2} g t^2 \Rightarrow y = -\frac{9,8}{2} t^2 = -4,9 t^2$$

$$\Rightarrow y_1 = -4,9 \cdot (0,3)^2 = -0,441 \text{ m}$$

$$y_2 = -4,9 \cdot (0,2)^2 = -0,196 \text{ m}$$

$$y_{cm} = \frac{y_1 \cdot m_1 + y_2 \cdot m_2}{m_1 + m_2} = \frac{-0,441 \cdot m - 0,196 \cdot 2m}{m + 2m} = -0,28 \text{ m}$$

$$\Rightarrow d = 28 \text{ cm}$$



b)

$$M \cdot \vec{v}_{cm} = m_1 \cdot \vec{v}_1 + m_2 \cdot \vec{v}_2 \text{ como o movimento é na vertical}$$

$$\Rightarrow M \cdot v_{cm} = m_1 \cdot v_1 + m_2 \cdot v_2$$

$$v = v_0 - g t \Rightarrow v = -g t \Rightarrow v_1 = -9,8 \cdot 0,3 = -2,94 \text{ m/s} \quad e \quad v_2 = -9,8 \cdot 0,2 = -1,96 \text{ m/s}$$

$$\Rightarrow (m + 2m) v_{cm} = -2,94 m - 1,96 \cdot 2m \Rightarrow v_{cm} = -2,29 \text{ m/s}$$

7.

$$\sum F_{ext} = 0 \Rightarrow c_m \rightarrow \text{Permanece em repouso}$$

$$\Rightarrow x_{cm} = x'_{cm}$$

$$\Rightarrow \frac{x_r m_r + x_c m_c + x_b m_b}{m_r + m_c + m_b} = \frac{x'_r m_r + x'_c m_c + x'_b m_b}{m_r + m_c + m_b}$$

$$\Rightarrow 0,4.80 + 3,4 m_c + 1,9.30 = 3.80 + 0. m_c + 1,5.30$$

$$\Rightarrow m_c = 57,65 \text{ kg}$$

8.

$$m_c = 4,5 \text{ kg}, m_b = 18 \text{ kg}$$

$$\sum \vec{F}_{ext} = 0 \Rightarrow x_{cm} \rightarrow \text{permanece em repouso}$$

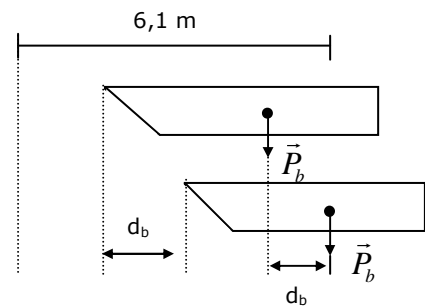
$$\Rightarrow x_{cm} = x'_{cm}$$

$$\Rightarrow \frac{m_c \cdot 6,1 + m_b \cdot d}{m_c + m_b} = \frac{m_c (6,1 - 2,4 + d_b) + m_b (d + d_b)}{m_c + m_b}$$

$$\Rightarrow m_c \cdot 6,1 + m_b \cdot d = m_c \cdot 6,1 - m_c \cdot 2,4 + m_c \cdot d_b + m_b \cdot d + m_b \cdot d_b$$

$$\Rightarrow 0 = -m_c \cdot 2,4 + (m_c + m_b) \cdot d_b \Rightarrow 0 = -4,5 \cdot 2,4 + (4,5 + 18) \cdot d_b \Rightarrow d_b = 0,48 \text{ m}$$

$$x'_c = 6,1 - 2,4 + 0,48 \Rightarrow x'_c = 4,18 \text{ m}$$



9.

$$m_f = 816 \text{ kg}, m_c = 2650 \text{ kg}, v_c = 16 \text{ km/h}$$

$$\text{a) } P_f = P_c \Rightarrow m_f \cdot v_f = m_c \cdot v_c \Rightarrow 816 \cdot v_f = 2650 \cdot 16 \Rightarrow v_f = 51,96 \text{ km/h}$$

$$\text{b) } E_{kf} = E_{kc} \Rightarrow \frac{1}{2} m_f \cdot v_f^2 = \frac{1}{2} m_c \cdot v_c^2 \Rightarrow 816 \cdot v_f^2 = 2650 \cdot 16^2 \Rightarrow v_f = 28,83 \text{ km/h}$$

10.

$$m = 80 \text{ kg}, m_c = 1600 \text{ kg}, v_c = 1,2 \text{ km/h}$$

$$P = P_c \Rightarrow m \cdot v = m_c \cdot v_c \Rightarrow 80 \cdot v = 1600 \cdot 1,2 \Rightarrow v = 24 \text{ km/h}$$

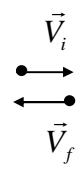
11.

Considerando:

$$m = 0,70 \text{ kg}, \vec{v} = 5 \text{ m/s} \hat{i}, \vec{v}_f = -2 \text{ m/s} \hat{i}$$

$$\Delta \vec{P} = \vec{P}_f - \vec{P}_i = 0,7 \cdot (-2) \hat{i} - 0,7 \cdot 5 \hat{i} = -(4,9 \text{ kg} \cdot \text{m/s}) \hat{i}$$

$$\Rightarrow \Delta P = 4,9 \text{ kg} \cdot \text{m/s}$$



12.

$$\vec{r} = (3500 - 160t)\hat{i} + 2700\hat{j} + 300\hat{k}, \quad m = 250 \text{ kg}$$

a)

$$\vec{P} = m\vec{v}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = -(160 \text{ m/s})\hat{i} \rightarrow \vec{P} = 250 \cdot (-160\hat{i}) \Rightarrow \vec{P} = -4 \cdot 10^4 \text{ kg} \cdot \text{m/s}$$

b) na direção  $-x \rightarrow$  oeste

c)

$$\vec{F}_R = m\vec{a}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = 0 \Rightarrow \vec{F}_R = 0$$

13.

$$m_h = 91 \text{ kg}, \quad m_p = 68 \text{ g} = 68 \cdot 10^{-3} \text{ kg}, \quad v_{FP} = 4 \text{ m/s}, \quad V_{ih} = V_{ip} = 0$$

com o movimento é um uma dimensão,

$$\sum \vec{P}_i = \sum \vec{P}_F \Rightarrow m_h \cdot v_{Fh} + m_p \cdot v_{Fp} = 0 \Rightarrow 91 \cdot V_{hF} + 68 \cdot 10^{-3} \cdot 4 = 0$$

$$\Rightarrow V_{Fh} = -3 \cdot 10^{-3} \text{ m/s} \text{ (sentido contrário ao da pedra)}$$

14.

$$m_1 = 1 \text{ kg} \text{ e } m_2 = 3 \text{ kg}, \quad V_1 = 1,7 \text{ m/s}$$

$$v_{i1} = v_{i2} = 0$$

$$\sum \vec{P}_i = \sum \vec{P}_F \Rightarrow m_1 v_1 + m_2 v_2 = 0 \Rightarrow 1 \cdot 1,7 + 3 \cdot v_2 = 0 \Rightarrow v_2 = -0,57 \text{ m/s}$$

15.

$$\vec{v}_i = (-0,4 \text{ m/s})\hat{i}, \quad m_A = 0,5 \text{ kg}, \quad \vec{v}_{FA} = ?, \quad m_B = 0,6 \text{ kg}, \quad \vec{v}_{FB} = (0,2 \text{ m/s})\hat{i}$$

$$m_C = 0,2 \text{ kg}, \quad \vec{v}_{FC} = (0,3 \text{ m/s})\hat{i}, \quad m = m_A + m_B + m_C = 1,3 \text{ kg}$$

$$\sum \vec{P}_i = \sum \vec{P}_F \Rightarrow m \cdot \vec{v}_i = m_A \cdot \vec{v}_{FA} + m_B \cdot \vec{v}_{FB} + m_C \cdot \vec{v}_{FC}$$

$$\Rightarrow 1,3 \cdot (0,4\hat{i}) = 0,5 \cdot \vec{v}_{FA} + 0,6 \cdot 0,2\hat{i} + 0,2 \cdot 0,3\hat{i} \Rightarrow \vec{v}_{FA} = -(1,4 \text{ m/s})\hat{i}$$

16.

a) Supondo:

$$\vec{P}_{FC} = (1,2 \cdot 10^{-22} \text{ kg.m/s}) \hat{i} \quad e \quad \vec{P}_{FN} = (6,4 \cdot 10^{-23} \text{ kg.m/s}) \hat{j}, \quad \vec{P}_{FN} = ?$$

$$\sum \vec{P}_i = \sum \vec{P}_F \Rightarrow \vec{P}_{FN} + \vec{P}_{Fe} + \vec{P}_{Fe} = 0 \Rightarrow \vec{P}_{iN} + (1,2 \cdot 10^{-22} \text{ kg.m/s}) \hat{i} + (6,4 \cdot 10^{-23} \text{ kg.m/s}) \hat{j} = 0$$

$$\Rightarrow \vec{P}_{FN} = -(1,2 \cdot 10^{-22} \text{ kg.m/s}) \hat{i} - (6,4 \cdot 10^{-23} \text{ kg.m/s}) \hat{j}$$

$$\Rightarrow P_{fN} = \sqrt{(1,2 \cdot 10^{-22})^2 + (6,4 \cdot 10^{-23})^2} = 1,36 \cdot 10^{-22} \text{ kg.m/s}$$

b)

$$\text{tg} \theta = \frac{6,4 \cdot 10^{-33}}{1,2 \cdot 10^{-22}} \Rightarrow \theta = 28^\circ$$

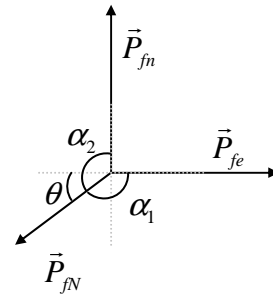
$$\alpha_1 = 180^\circ - \theta = 180^\circ - 28^\circ = 152^\circ$$

c)  $\alpha_2 = 90^\circ + \theta = 90^\circ + 28^\circ = 118^\circ$

d)

$$m = 5,8 \cdot 10^{-26} \text{ kg} \quad P_{FN} = m_N \cdot v_{FN} \Rightarrow 1,36 \cdot 10^{-22} = 5,8 \cdot 10^{-26} v_{FN} \Rightarrow v_{FN} = 2345 \text{ m/s}$$

$$E_k = \frac{1}{2} m_N v_N^2 = \frac{1}{2} \cdot 5,8 \cdot 10^{-26} (2345)^2 \Rightarrow E_k = 1,6 \cdot 10^{-19} \text{ J}$$



17.

$$m = 20 \text{ kg}, \quad \vec{v}_i = (200 \text{ m/s}) \hat{i}, \quad m_1 = 10 \text{ kg}, \quad \vec{v}_{f1} = (100 \text{ m/s}) \hat{j}$$

$$m_2 = 4 \text{ kg}, \quad \vec{v}_2 = -(500 \text{ m/s}) \hat{i}$$

a)

$$m_3 = 6 \text{ kg} \quad \vec{v}_{F3} = ?$$

$$\sum \vec{F}_{ext} = 0 \Rightarrow \sum \vec{P}_i = \sum \vec{P}_F \Rightarrow M \vec{v}_i = m_1 \vec{v}_{F1} + m_2 \vec{v}_{F2} + m_3 \vec{v}_{F3}$$

$$\Rightarrow 20 \cdot 200 \hat{i} = 10 \cdot 100 \hat{j} - 4 \cdot 500 \hat{i} + 6 \cdot \vec{v}_{F3} \Rightarrow \vec{v}_{F3} = \frac{(20 \cdot 200 + 4 \cdot 500) \hat{i} - 1000 \hat{j}}{6} = 1000 \hat{i} - 166,67 \hat{j}$$

$$v_{F3} = 1013,79 \text{ m/s}$$

$$\text{tg} \theta = \frac{166,67}{1000} \Rightarrow \theta = 9,46^\circ$$

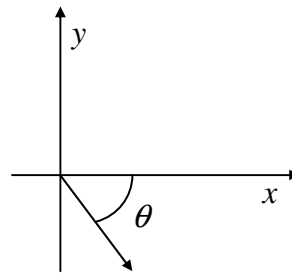
b)

$$E_{ki} = \frac{1}{2} M v_i^2 = \frac{1}{2} 20 \cdot 200^2 = 4 \cdot 10^5 \text{ J}$$

$$E_{kf} = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + \frac{1}{2} m_3 v_3^2 = \frac{1}{2} (10 \cdot 100^2 + 4 \cdot 500^2 + 6 \cdot 1013,79^2)$$

$$\Rightarrow E_{kf} = 3,63 \cdot 10^6 \text{ J}$$

$$\Delta E_k = E_{kf} - E_{ki} = 3,63 \cdot 10^6 - 4 \cdot 10^5 = 3,23 \cdot 10^6 \text{ J}$$



18.

$$v_i = 0$$

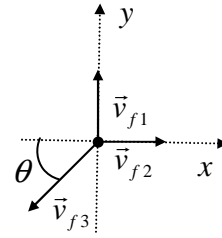
$$m_1 = m_2 = m, m_3 = 3m, \text{ supondo:}$$

$$\vec{v}_{f1} = (30m/s)\hat{i}, \quad \vec{v}_{f2} = (30m/s)\hat{j}$$

$$\sum \vec{F}_{ext} = 0 \Rightarrow \sum \vec{P}_i = \sum \vec{P}_f \Rightarrow m_1 \vec{v}_{f1} + m_2 \vec{v}_{f2} + m_3 \vec{v}_{f3} = 0$$

$$\Rightarrow m \cdot 30\hat{i} + m \cdot 30\hat{j} + 3m \vec{v}_{f3} = 0 \Rightarrow \vec{v}_{f3} = (-10\hat{i} + 10\hat{j})m/s$$

$$v_{f3} = \sqrt{10^2 + 10^2} = 14,14m/s \quad \text{tg}\theta = \frac{10}{10} \Rightarrow \theta = 45^\circ$$



19.

$$M_i = 6090\text{ kg}, v_i = 105\text{ m/s}, M_f = 6090 - 80 = 6010\text{ kg}$$

$$v_{rel} = 253\text{ m/s}$$

$$v_f - v_i = v_{rel} \ln\left(\frac{M_i}{M_f}\right) \Rightarrow v_f = 105 + 253 \ln\left(\frac{6090}{6010}\right) \rightarrow v_f = 108,34\text{ m/s}$$

20.

$$v_i = 6 \cdot 10^3\text{ m/s}, v_{rel} = 3 \cdot 10^3\text{ m/s}, M = 4 \cdot 10^4\text{ kg}, a = 2\text{ m/s}^2$$

$$\text{a) } E = M \cdot a = 4 \cdot 10^4 \cdot 2 = 8 \cdot 10^4\text{ N}$$

$$\text{b) } R = ? \quad v_{rel} \cdot R = M \cdot a \Rightarrow 3 \cdot 10^3 R = 4 \cdot 10^4 \cdot 2 \Rightarrow R = 26,67\text{ kg/s}$$

21.

$$M_i = 2,55 \cdot 10^5\text{ kg}, \quad M_c = 1,81 \cdot 10^5\text{ kg}, \quad \Delta t = 250\text{ s}, \quad v_i = 0$$

$$\frac{dm}{dt} = 480\text{ kg/s}, \quad v_{rel} = 3,27\text{ km/s}$$

$$\text{a) } E = v_{rel} \cdot R = v_{rel} \cdot \frac{dm}{dt} = 3,27 \cdot 10^3 \cdot 480 = 1,57 \cdot 10^6\text{ N}$$

$$\text{b) } m'_c = 480 \cdot 250 = 1,2 \cdot 10^5\text{ kg} \text{ (combustível consumido)}$$

$$\Rightarrow M_f = 2,55 \cdot 10^5 - 1,2 \cdot 10^5 = 1,35 \cdot 10^5\text{ kg}$$

$$\text{c) } v_f - v_i = v_{rel} \ln\left(\frac{M_i}{M_f}\right) \Rightarrow v_f = 3,27 \ln\left(\frac{2,55 \cdot 10^5}{1,35 \cdot 10^5}\right) = 2,08\text{ km/s}$$

22.

$$F_m = 50\text{ N} \quad m = 0,2\text{ kg} \rightarrow \text{movimento em uma dimensão.}$$

$$\Delta t = 10 \text{ ms} = 10^{-2} \text{ s}$$

$$j = \Delta P \rightarrow F_m \cdot \Delta t = m \cdot v_f - mv_i$$

$$\rightarrow 50 \cdot 10^{-2} = 0,2 \cdot v_f \rightarrow v_f = 2,5 \text{ m/s}$$

23.

$$m = 150 \text{ g} = 0,15 \text{ kg}, \quad v_i = 40 \text{ m/s}, \quad v_f = -60 \text{ m/s}, \quad \Delta t = 5 \cdot 10^{-3} \text{ s},$$

como o movimento é em uma dimensão

$$j = \Delta p \Rightarrow F_m \cdot \Delta t = mv_f - mv_i \rightarrow F_m \cdot 5 \cdot 10^{-3} = 0,15(-60 - 40) \rightarrow F_m = -3000 \text{ N}$$

$$\rightarrow |F_m| = 3000 \text{ N}$$

25.

Movimento em uma dimensão

$$m = 1,2 \text{ kg}, \quad v_i = -25 \text{ m/s}, \quad v_f = 10 \text{ m/s}, \quad \Delta t = 0,02 \text{ s}$$

$$\text{a) } j = \Delta p = mv_f - mv_i = 1,2 \cdot 10 - 1,2 \cdot (-25) = 42 \text{ N}\cdot\text{s}$$

$$\text{b) } j = F_m \cdot \Delta t \Rightarrow 42 = F_m \cdot 0,02 \Rightarrow F_m = 2100 \text{ N}$$

26.

$$m = 1400 \text{ kg}, \quad \vec{v}_i = 5,3 \text{ m/s } \hat{j}, \quad \Delta t_1 = 4,6 \text{ s}, \quad \Delta t_2 = 35 \text{ ms} = 0,35 \text{ s}$$

a)

$$\vec{v}_2 = 5,3 \text{ m/s } \hat{i}$$

$$\vec{j}_1 = \Delta \vec{p} = \vec{P}_f - \vec{P}_i = m\vec{v}_f - m\vec{v}_i = 1400 \cdot (5,3\hat{i} - 5,3\hat{j})$$

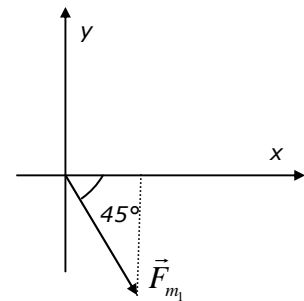
$$\Rightarrow \vec{j}_1 = (7420\hat{i} - 7420\hat{j}) \text{ N}\cdot\text{s}$$

$$\text{b) } \vec{j}_2 = \Delta \vec{p} = \vec{p}_f - \vec{P}_i = -m\vec{v}_2 = -1400 \cdot 5,3\hat{i} \Rightarrow \vec{j}_2 = -(7420\hat{i}) \text{ N}\cdot\text{s}$$

$$\text{c) } j_1 = F_{m_1} \cdot \Delta t_1 = \sqrt{7420^2 + 7420^2} = F_{m_1} \cdot 4,6 \Rightarrow F_{m_1} = 2,28 \cdot 10^3 \text{ N}$$

$$\text{d) } j_2 = F_{m_2} \cdot \Delta t_2 \Rightarrow 7420 = F_{m_2} \cdot 0,35 \Rightarrow F_{m_2} = 2,12 \cdot 10^4 \text{ N}$$

e) A direção de  $\vec{F}_{m_1}$  é a mesma de  $\vec{j}_1$



27.

Movimento em uma dimensão



$$m = 10\text{kg}, F_i = 0, F_f = 50\text{N}, \Delta t = 4\text{s}, v_i = 0$$

$$F = At, p/t = 4 \Rightarrow F = 50 \Rightarrow 50 = A \cdot 4 \Rightarrow A = 12,5\text{N/s} \Rightarrow F = 12,5t$$

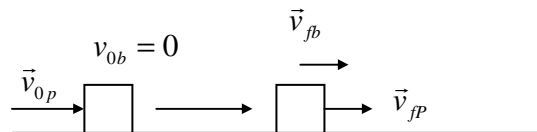
$$j = \Delta p \Rightarrow \int_0^4 F dt = mv_f - mv_i \Rightarrow \int_0^4 12,5t dt = mv_f$$

$$\Rightarrow 12,5 \left. \frac{t^2}{2} \right|_0^4 = mv_f$$

$$\Rightarrow 12,5 \cdot \frac{4^2}{2} = 10 \cdot v_f \Rightarrow v_f = 10\text{m/s}$$

28.

O movimento é em uma dimensão



$$m_p = 5,2\text{g} = 5,2 \cdot 10^{-3}\text{kg}, v_{ip} = 672\text{m/s}, m_b = 700\text{g} = 0,7\text{kg}$$

$$v_{ib} = 0, v_{fp} = 428\text{m/s}$$

a)  $v_{fb} = ?$

$$\sum F_{ex} = 0 \Rightarrow \sum P_i = \sum P_f \Rightarrow m_p v_{0p} + m_b v_{0b} = m_p v_{fp} + m_b v_{fb}$$

$$\Rightarrow 5,2 \cdot 10^{-3} \cdot 672 = 5,2 \cdot 10^{-3} \cdot 428 + 0,7 \cdot v_{fb} \Rightarrow v_{fb} = 1,81\text{ m/s}$$

b)

$$M \cdot V_{cm} = m_p v_{ip} + m_b v_{ib}$$

$$\Rightarrow (5,2 \cdot 10^{-3} + 0,7) V_{cm} = 5,2 \cdot 10^{-3} \cdot 672 \Rightarrow V_{cm} = 4,9\text{ m/s}$$

29.

$$m_t = 6\text{kg}, v_{it} = 9\text{m/s}, m_p = 12\text{kg}, v_{ip} = 0$$

$$\sum F_{ex} = 0 \Rightarrow \sum \vec{P}_i = \sum \vec{P}_f \Rightarrow m_t v_{it} + m_p v_{ip} = (m_t + m_p) v_f$$

$$\Rightarrow 6 \cdot 9 = (6 + 12) v_f \Rightarrow v_f = 3\text{m/s}$$

30.

O movimento é em uma dimensão

$$m_p = 4,5\text{g} = 4,5 \cdot 10^{-3}\text{kg}, m_b = 2,4\text{kg}, v_{ib} = 0, \mu = 0,2, d = 1,8\text{m}$$

a)

$$\Delta E = w_{fat} \Rightarrow u_f + E_{kf} - (\mu_i + E_{ki}) = -\mu_c \cdot n \cdot d$$

$$\Rightarrow -\frac{1}{2} (m_p + m_b) v_i^2 = -\mu_c \cdot (m_p + m_b) \cdot g \cdot d \Rightarrow \frac{v_i^2}{2} = 0,2 \cdot 9,8 \cdot 1,8$$

$$\Rightarrow v_i = 2,7 \text{ m/s}$$

b)

$$\sum \vec{P}_i = \sum \vec{P}_f \Rightarrow m_p \cdot v'_{ip} + m_b \cdot v'_{ib} = (m_p + m_b)v_i$$

$$\Rightarrow 4,5 \cdot 10^{-3} \cdot v'_{ip} = (4,5 \cdot 10^{-3} + 2,4) \cdot 2,7$$

$$\Rightarrow v'_{ip} = 1442,7 \text{ m/s}$$

31.

$$m_A = 1100 \text{ kg}, m_g = 1400 \text{ kg}, \mu_k = 0,13, d_A = 8,2 \text{ m}, d_B = 6,1 \text{ m}$$

a)

$$v_{iA} = ?$$

$$\Delta E = w_{fat} \Rightarrow u_f + E_{kf} - (u_i + E_{ki}) = -\mu_c \cdot mgd \Rightarrow \frac{1}{2}mv_i^2 = \mu_c \cdot mgd$$

$$\Rightarrow v_i = \sqrt{2 \cdot \mu_c \cdot gd} \Rightarrow v_{iA} = \sqrt{2 \cdot 0,13 \cdot 9,8 \cdot 8,2} \Rightarrow v_{iA} = 4,6 \text{ m/s}$$

$$\text{b) } v_{iB} = \sqrt{2 \cdot \mu_c \cdot g \cdot d_B} = \sqrt{2 \cdot 0,13 \cdot 9,8 \cdot 6,1} \Rightarrow v_{iB} = 3,9 \text{ m/s}$$

c)

imediatamente antes e após a colisão, temos que:

$$\sum \vec{P}_i = \sum \vec{P}_f \Rightarrow m_A \cdot v'_{iA} + m_B \cdot v'_{iB} = m_A v_{iA} + m_B v_{iB}$$

$$\Rightarrow 1400 \cdot v'_{iB} = 1100 \cdot 4,6 + 1400 \cdot 3,9 \Rightarrow v'_{iB} = 7,5 \text{ m/s}$$

32.

$$m_1 = 2 \text{ kg}, v_{1i} = 10 \text{ m/s}, m_2 = 5 \text{ kg}, v_{2i} = 3 \text{ m/s}, k = 1120 \text{ N/m}$$

$$\sum \vec{P}_i = \sum \vec{P}_f \Rightarrow m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f$$

$$\Rightarrow 2 \cdot 10 + 5 \cdot 3 = (2 + 5) v_f \Rightarrow v_f = 5 \text{ m/s}$$

a energia mecânica do sistema se conserva

$$E_i = E_f \Rightarrow E_{ki} + u_i = E_{kf} + u_f$$

$$\Rightarrow \frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} (m_1 + m_2) v_f^2 + \frac{1}{2} kx^2$$

$$\Rightarrow 2 \cdot 10^2 + 5 \cdot 3^2 = (2 + 5) \cdot 5^2 + 1120 \cdot x^2 \Rightarrow x = 0,25 \text{ m}$$

33.

$$m_1 = 340 \text{ g} = 0,34 \text{ kg}, v_{1i} = 1,2 \text{ m/s}, v_{2i} = 0, v_{1f} = 0,66 \text{ m/s}$$

a) e b)

$$\begin{aligned}\sum \vec{P}_i &= \sum \vec{P}_f \Rightarrow m_1 \cdot v_{1i} + m_2 v_{2i} = m_2 v_{2f} + m_1 v_{1f} \\ \Rightarrow 0,34 \cdot 1,2 &= 0,34 \cdot 0,66 + m_2 \cdot v_{2f} \\ \Rightarrow 0,184 &= m_2 v_{2f}\end{aligned}$$

na colisão elástica a energia cinética do sistema se conserva

$$\begin{aligned}E_{k_{1i}} + E_{k_{2i}} &= E_{k_{1f}} + E_{k_{2f}} \Rightarrow \frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 \\ \Rightarrow 0,34 \cdot (1,2)^2 &= 0,34 \cdot (0,66)^2 + m_2 v_{2f}^2 \Rightarrow 0,341 = m_2 v_{2f}^2 \\ \Rightarrow 0,341 &= m_2 \cdot v_{2f} \cdot v_{2f} \Rightarrow 0,341 = 0,184 \cdot v_{2f} \Rightarrow v_{2f} = 1,86 \text{ m/s} \\ \Rightarrow m_2 &= \frac{0,184}{1,86} = 0,099 \text{ kg}\end{aligned}$$

$$c) v_{cm} = \frac{v_{1i} m_1 + v_{2i} m_2}{m_1 + m_2} = \frac{0,34 \cdot 1,2}{0,34 + 0,099} = 0,93 \text{ m/s}$$

34.

$$m_1 = 0,5 \text{ kg}, L = 70 \text{ cm} = 0,7 \text{ m}, m_2 = 2,5 \text{ kg}, v_{2i} = 0$$

Cálculo da velocidade da bola imediatamente antes da colisão. Considerando apenas a bola, temos que:

$$\begin{aligned}E_i &= E_f \Rightarrow u_i + \frac{1}{2} m v_i^2 = u_f + \frac{1}{2} m v_f^2 \Rightarrow mgh = \frac{1}{2} m v_f^2 \\ \Rightarrow v_f &= \sqrt{2gL} = \sqrt{2 \cdot 9,8 \cdot 0,7} = 3,7 \text{ m/s}\end{aligned}$$

Considerando a bola e o bloco, imediatamente antes e depois da colisão, temos que:

$$\begin{aligned}\sum \vec{P}_i &= \sum \vec{P}_f \Rightarrow m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} \\ \Rightarrow 0,5 \cdot 3,7 &= 0,5 \cdot v_{1f} + 2,5 \cdot v_{2f} \Rightarrow 3,7 = v_{1f} + 5v_{2f} \Rightarrow v_{1f} = 3,7 - 5v_{2f}\end{aligned}$$

Na colisão elástica a energia cinética se conserva

$$\begin{aligned}\Rightarrow \frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 &= \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 \\ \Rightarrow 0,5 \cdot (3,7)^2 &= 0,5 \cdot v_{1f}^2 + 2,5 v_{2f}^2 \Rightarrow 13,69 = v_{1f}^2 + 5v_{2f}^2 \\ \Rightarrow 13,69 &= (3,7 - 5v_{2f})^2 + 5v_{2f}^2 \Rightarrow v_{2f} = 1,23 \text{ m/s} \text{ e } v_{1f} = 3,7 - 5v_{2f} = -2,45 \text{ m/s}\end{aligned}$$

35.

$$\begin{aligned}m_A &= m_B = 2 \text{ kg}, \vec{v}_{iA} = 15\hat{i} + 30\hat{j}, \vec{v}_{iB} = -10\hat{i} + 5\hat{j} \\ \vec{v}_{fA} &= -5\hat{i} + 20\hat{j}\end{aligned}$$

a)

$$\begin{aligned}\sum \vec{P}_i &= \sum \vec{P}_f \Rightarrow m_A \vec{v}_{iA} + m_B \vec{v}_{iB} = m_A \vec{v}_{fA} + m_B \vec{v}_{fB} \\ \Rightarrow 15\hat{i} + 30\hat{j} - 10\hat{i} + 5\hat{j} &= -5\hat{i} + 20\hat{j} \cdot \vec{v}_{fB} \\ \Rightarrow \vec{v}_{fB} &= (10\hat{i} + 15\hat{j})m/s\end{aligned}$$

b)

$$E_{ki} = \frac{1}{2}m_A v_{Ai}^2 + \frac{1}{2}m_B v_{Bi}^2$$

$$v_{Ai} = \sqrt{15^2 + 30^2} = 33,54m/s, \quad v_{Bi} = \sqrt{10^2 + 5^2} = 11,18m/s, \quad v_{Af} = \sqrt{5^2 + 20^2} = 20,61m/s$$

$$v_{Bf} = \sqrt{10^2 + 15^2} = 18,03m/s$$

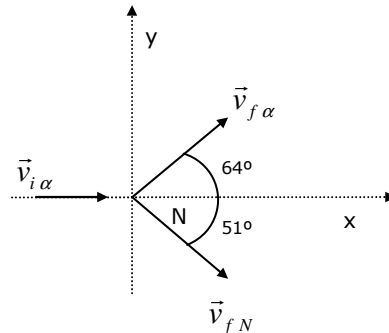
$$\Rightarrow E_{ki} = \frac{1}{2} \cdot 2 \cdot (33,54)^2 + \frac{1}{2} \cdot 2 \cdot (11,18)^2 = 1249,92J$$

$$E_{kf} = \frac{1}{2} \cdot 2 \cdot (20,61)^2 + \frac{1}{2} \cdot 2 \cdot (18,03)^2 = 749,85J$$

$$\Delta E_k = E_{kf} - E_{ki} = 749,85 - 1249,92 = -500,07J$$

36.

$$m_\alpha = 4u, \quad m_N = 16u, \quad v_{iN} = 0, \quad v_{fN} = 1,2 \cdot 10^5 m/s$$



a)

$$\sum \vec{P}_i = \sum \vec{P}_f \Rightarrow m_\alpha \vec{v}_{i\alpha} + m_N \vec{v}_{Ni} = m_\alpha \vec{v}_{F\alpha} + m_N \vec{v}_{FN}$$

$$\Rightarrow 4 \cdot v_{i\alpha} \hat{i} = 4(v_{F\alpha} \cos 64^\circ \hat{i} + v_{F\alpha} \sin 64^\circ \hat{j}) + 16(1,2 \cdot 10^5 \cos 51^\circ \hat{i} - 1,2 \cdot 10^5 \sin 51^\circ \hat{j})$$

$$v_{i\alpha} \hat{i} = (v_{F\alpha} \cos 64^\circ + 3,02 \cdot 10^5) \hat{i} + (v_{F\alpha} \sin 64^\circ - 3,73 \cdot 10^5) \hat{j}$$

$$\Rightarrow v_{F\alpha} \sin 64^\circ - 3,73 \cdot 10^5 = 0 \Rightarrow v_{F\alpha} = 4,15 \cdot 10^5 m/s$$

b)

$$v_{i\alpha} = v_{F\alpha} \cos 64^\circ + 3,02 \cdot 10^5 = 4,15 \cdot 10^5 \cdot \cos 64^\circ + 3,02 \cdot 10^5 \Rightarrow v_{i\alpha} = 4,84 \cdot 10^5 m/s$$

37.

$$m_1 = m_2 = m, \quad v_{i2} = 0, \quad v_{1f} = 3,5m/s, \quad v_{2f} = 2m/s$$

a)

$$\begin{aligned}\sum \vec{P}_i &= \sum \vec{P}_f \Rightarrow m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f} \\ \Rightarrow v_{1i} \hat{i} &= 3,5 \cos 22^\circ \hat{i} + 3,5 \sin 22^\circ \hat{j} + 2 \cos \theta \hat{i} - 2 \sin \theta \hat{j} \\ \Rightarrow v_{1i} \hat{i} &= (3,5 \cos 22^\circ + 2 \cos \theta) \hat{i} + (3,5 \sin 22^\circ - 2 \sin \theta) \hat{j} \\ \Rightarrow 3,5 \sin 22^\circ - 2 \sin \theta &= 0 \Rightarrow \theta = 40,96^\circ\end{aligned}$$

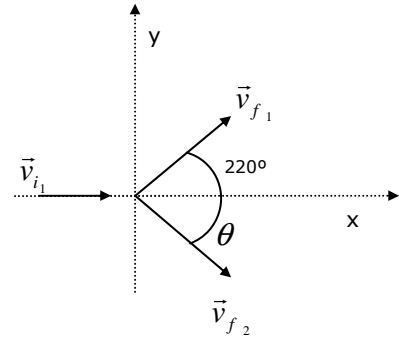
b)  $v_{1i} = 3,5 \cos 22^\circ + 2 \cos(40,96^\circ) \Rightarrow v_{1i} = 4,75 \text{ m/s}$

c)

$$E_{ki} = \frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m (4,75)^2 = 11,28 \text{ m}$$

$$E_{kf} = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 = \frac{1}{2} m (3,5)^2 + \frac{1}{2} m \cdot 2^2 = 8,125 \text{ m}$$

$$E_{ki} \neq E_{kf} \Rightarrow \text{n\~{a}o}$$



38.

$$m_1 = m_2 = m, \quad v_{1i} = v_{2i} = v_i, \quad v_f = \frac{v_i}{2}$$

$$\sum \vec{P}_i = \sum \vec{P}_f$$

$$\Rightarrow m(v_i \cos \theta_1 \hat{i} - v_i \sin \theta_1 \hat{j}) + m(v_i \cos \theta_2 \hat{i} + v_i \sin \theta_2 \hat{j}) = 2m v_f \hat{i}$$

$$\Rightarrow v_i (\cos \theta_1 \hat{i} - \sin \theta_1 \hat{j}) + v_i (\cos \theta_2 \hat{i} + \sin \theta_2 \hat{j}) = 2 \frac{v_i}{2} \hat{i}$$

$$\Rightarrow (\cos \theta_1 + \cos \theta_2) \hat{i} + (-\sin \theta_1 + \sin \theta_2) \hat{j} = 1 \hat{i}$$

$$\Rightarrow -\sin \theta_1 + \sin \theta_2 = 0 \Rightarrow \sin \theta_1 = \sin \theta_2 \Rightarrow \theta_1 = \theta_2$$

$$\cos \theta_1 + \cos \theta_2 = 1 \Rightarrow \cos \theta_1 + \cos \theta_1 = 1 \Rightarrow 2 \cos \theta_1 = 1 = \theta_1 = 60^\circ$$

$$\Rightarrow \alpha = 2\theta_1 = 2 \cdot 60 \Rightarrow \alpha = 120^\circ$$

