PUC Goiás / ENG / TEORIA-II / A01 / CHAER / 2010.2

P1/N1 - em 20/9/2010

(duração: 90' = 1,5 horas)

. . . .Nome:

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1 – VIGAS CONTÍNUAS – MÉTODO DOS DESLOCAMENTOS

Para a viga (**Fig.01**), com **EI = cte**, pede-se:

a) [4.0] Encontrar os **Deslocamentos incógnitos**;
b) [2.0] Determinar os **Momentos Fletores Finais** (AB,BA,BC,CB,CD,DC,DE,ED)**;**
c) [1.0] Traçar o **DMF** para a barra **CD**;
d) [2.0] Traçar o **DEC** para a barra **BC**;
e) [1.0] Determinar a Reação no apoio **B**.

sugestão:

- eliminar o balanço, transferindo as ações do balanço para o apoio;
- considerar o S.P. reduzido (sem as rotações nos nós extremos apoiados).

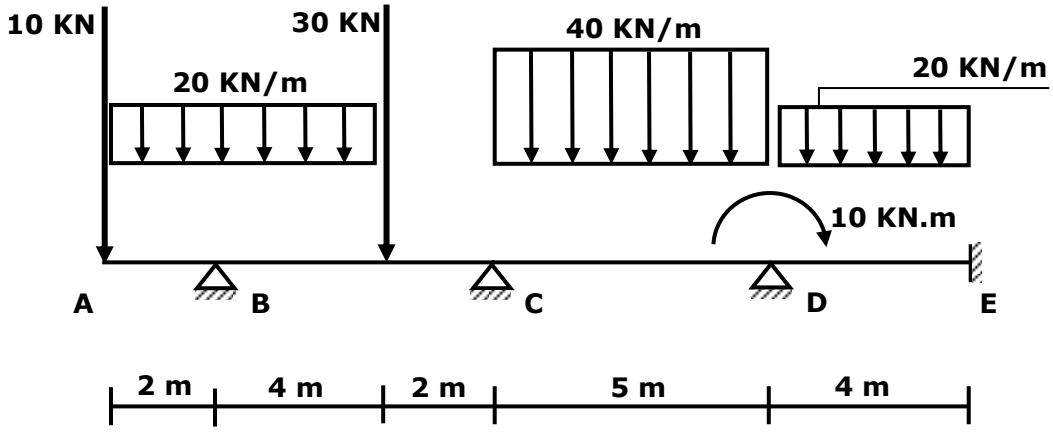


Figura 01

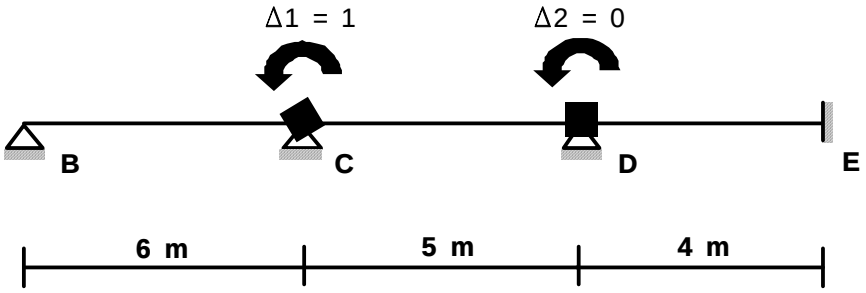
Desejo-lhe um bom desempenho!

CHAER

PONTIFÍCIA UNIVERSIDADE CATÓLICA DE GOIÁS
DEPARTAMENTO DE ENGENHARIA
TEORIA DAS ESTRUTURAS II / 2010.2
Prof. Alberto Vilela CHAER
P1 / N1 - PROVA D - EM 20.9.2010 - RESOLUÇÃO QUESTÃO 1

1 - Efeito E1 (Δ1=1; Δ2=0; carregamento nulo)

E1 (Δ1 = 1)



1.1 - Tramo **BC** (APOIO-ENGASTE)

$L := 6 \qquad I := 1 \qquad E := 1$

$M\_BC\_1 := 0 \qquad M\_CB\_1 := \frac{3 \cdot E \cdot I}{L} \qquad M\_CB\_1 = 0.500$

1.2 - Tramo **CD** (ENGASTE-ENGASTE)

$L := 5 \qquad I := 1 \qquad E := 1$

$M\_CD\_1 := \frac{4 \cdot E \cdot I}{L} \qquad M\_CD\_1 = 0.8 \qquad M\_DC\_1 := \frac{2 \cdot E \cdot I}{L} \qquad M\_DC\_1 = 0.400$

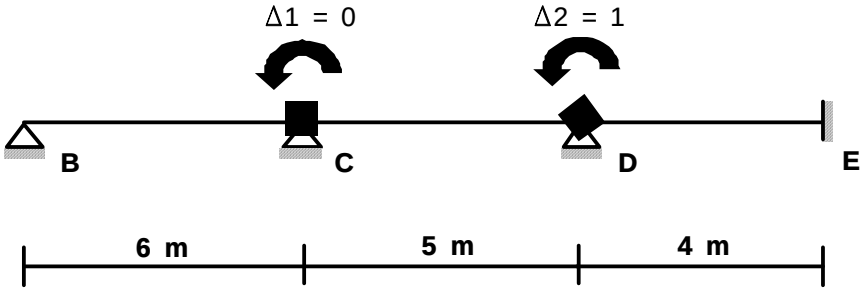
1.3 - Tramo **DE** (ENGASTE-ENGASTE)

$L := 4 \qquad I := 1 \qquad E := 1$

$M\_DE\_1 := 0 \qquad M\_ED\_1 := 0$

2 - Efeito E2 (Δ1=0; Δ2=1; carregamento nulo)

E2 (Δ2 = 1)



2.1 - Tramo **BC** (APOIO-ENGASTE)

$L := 6 \qquad I := 1 \qquad E := 1$

$M\_BC\_2 := 0 \qquad M\_CB\_2 := 0$

2.2 - Tramo **CD** (ENGASTE-ENGASTE)

$L := 5 \qquad I := 1 \qquad E := 1$

$M\_CD\_2 := \frac{2 \cdot E \cdot I}{L} \qquad M\_CD\_2 = 0.4 \qquad M\_DC\_2 := \frac{4 \cdot E \cdot I}{L} \qquad M\_DC\_2 = 0.800$

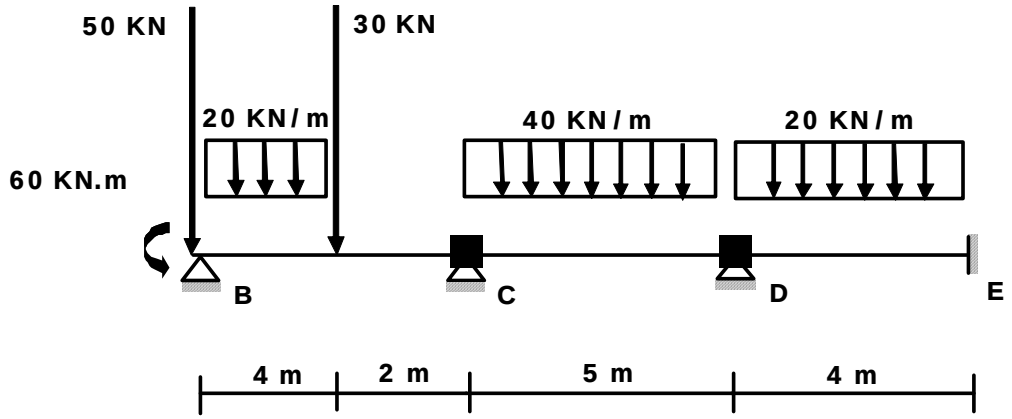
2.3 - Tramo **DE** (ENGASTE-ENGASTE)

$L := 4 \qquad I := 1 \qquad E := 1$

$M\_DE\_2 := \frac{4 \cdot E \cdot I}{L} \qquad M\_DE\_2 = 1.000 \qquad M\_ED\_2 := \frac{2 \cdot E \cdot I}{L} \qquad M\_ED\_2 = 0.500$

3 - Efeito E0 (carregamento; Δ1=0; Δ2=0)

E0



3.1 - **Tramo BC (APOIO-ENGASTE)**

$L := 6$

3.1.1 - Carga distribuída parcial (q)

$q := 20$ $c := 4$ $a := 2$ $b := 4$

$M\_BC\_q := 0$ $M\_BC\_q = 0$

$M\_CB\_q := -\frac{q \cdot a \cdot c}{8 \cdot L^2} \cdot \left[4 \cdot b \cdot (a + L) - c^2\right]$ $M\_CB\_q = -62.222$

3.1.2 - Carga concentrada (p)

$p := 30$ $a := 4$ $b := 2$

$M\_BC\_p := 0$ $M\_BC\_p = 0$

$M\_CB\_p := -\frac{p \cdot a \cdot b}{2 \cdot L^2} \cdot (L + a)$ $M\_CB\_p = -33.333$

3.1.3 - Momento do balanço (m)

$m := 10 \cdot 2 + \frac{20 \cdot 2^2}{2}$ $m = 60$

$M\_BC\_m := m$ $M\_BC\_m = 60$ $M\_CB\_m := \frac{m}{2}$ $M\_CB\_m = 30$

3.1.4 - Momentos acumulados

$M\_BC\_0 := M\_BC\_q + M\_BC\_p + M\_BC\_m \qquad M\_BC\_0 = 60$

$M\_CB\_0 := M\_CB\_q + M\_CB\_p + M\_CB\_m \qquad M\_CB\_0 = -65.556$

3.2 - **Tramo CD (ENGASTE-ENGASTE)**

$L := 5$

3.2.1 - Carga distribuída uniforme total (q)

$q := 40$

$M\_CD\_q := \frac{q \cdot L^2}{12} \qquad M\_CD\_q = 83.333 \qquad M\_DC\_q := -\frac{q \cdot L^2}{12} \qquad M\_DC\_q = -83.333$

3.2.2 - Momentos acumulados

$M\_CD\_0 := M\_CD\_q \qquad M\_CD\_0 = 83.333$

$M\_DC\_0 := M\_DC\_q \qquad M\_DC\_0 = -83.333$

3.3 - **Tramo DE (ENGASTE-ENGASTE)**

$L := 4$

3.3.1 - Carga distribuída uniforme total (q)

$q := 20$

$M\_DE\_q := \frac{q \cdot L^2}{12} \qquad M\_DE\_q = 26.667 \qquad M\_ED\_q := -\frac{q \cdot L^2}{12} \qquad M\_ED\_q = -26.667$

3.3.2 - Momentos acumulados

$M\_DE\_0 := M\_DE\_q \qquad M\_DE\_0 = 26.667$

$M\_ED\_0 := M\_ED\_q \qquad M\_ED\_0 = -26.667$

4 - Determinação dos coeficientes β

$\beta\_10 := M\_CB\_0 + M\_CD\_0 \qquad \beta\_10 = 17.778$

$\beta\_20 := M\_DC\_0 + M\_DE\_0 \qquad \beta\_20 = -56.667$

$\beta\_11 := M\_CB\_1 + M\_CD\_1 \qquad \beta\_11 = 1.3$

$\beta\_21 := M\_DC\_1 + M\_DE\_1 \qquad \beta\_21 = 0.4$

$\beta\_12 := M\_CB\_2 + M\_CD\_2 \qquad \beta\_12 = 0.4$

$\beta\_22 := M\_DC\_2 + M\_DE\_2 \qquad \beta\_22 = 1.8$

5 - Sistema de equações de equilíbrio

Given

$\beta\_10 + \beta\_11 \cdot \Delta 1 + \beta\_12 \cdot \Delta 2 = 0$

$\beta\_20 + \beta\_21 \cdot \Delta 1 + \beta\_22 \cdot \Delta 2 = -10$

$$\Delta := \text{Find}(\Delta 1, \Delta 2) \rightarrow \begin{pmatrix} \frac{-7600}{327} \\ \frac{30500}{981} \end{pmatrix}$$

$\Delta_1 = -23.242 \qquad \Delta_2 = 31.091$

6 - Momentos Finais

$M\_BC\_F := M\_BC\_0 + M\_BC\_1 \cdot \Delta_1 + M\_BC\_2 \cdot \Delta_2 \qquad M\_BC\_F = 60$

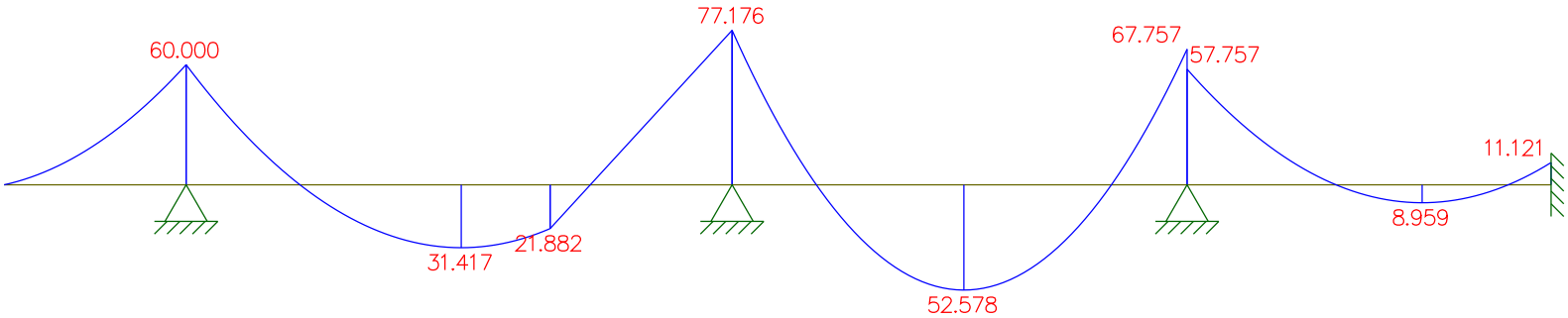
$M\_CB\_F := M\_CB\_0 + M\_CB\_1 \cdot \Delta_1 + M\_CB\_2 \cdot \Delta_2 \qquad M\_CB\_F = -77.176$

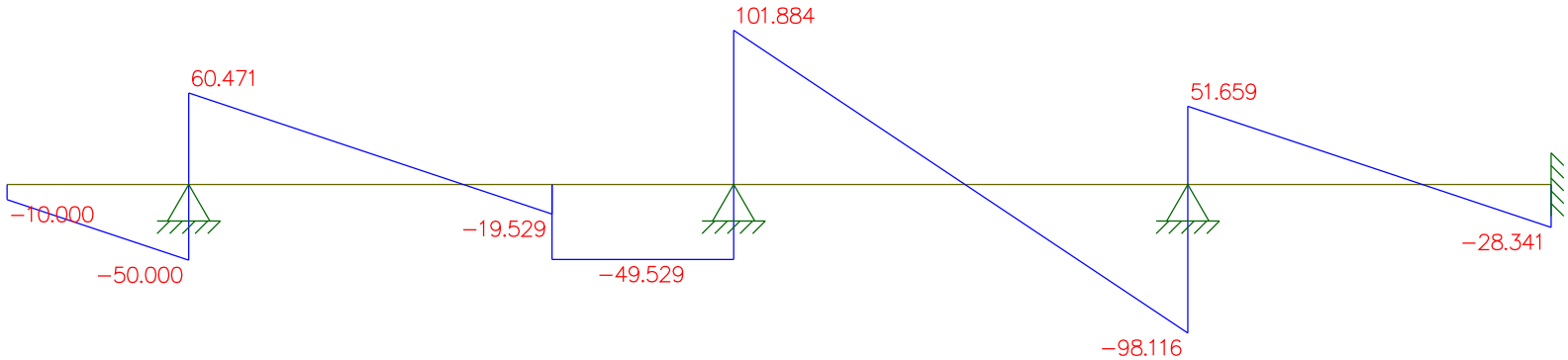
$M\_CD\_F := M\_CD\_0 + M\_CD\_1 \cdot \Delta_1 + M\_CD\_2 \cdot \Delta_2 \qquad M\_CD\_F = 77.176$

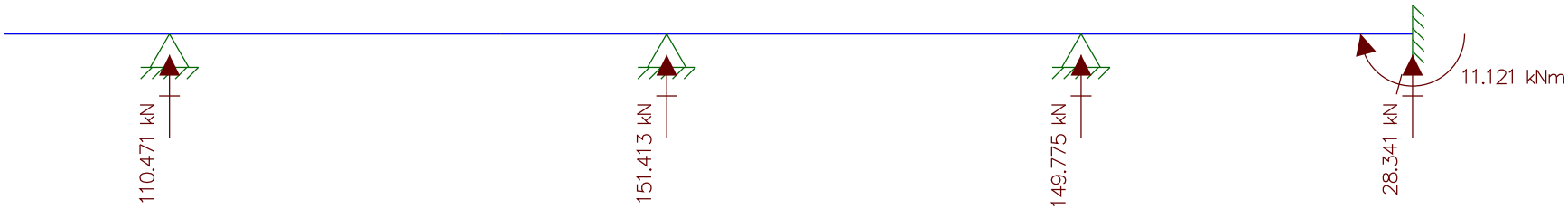
$M\_DC\_F := M\_DC\_0 + M\_DC\_1 \cdot \Delta_1 + M\_DC\_2 \cdot \Delta_2 \qquad M\_DC\_F = -67.757$

$M\_DE\_F := M\_DE\_0 + M\_DE\_1 \cdot \Delta_1 + M\_DE\_2 \cdot \Delta_2 \qquad M\_DE\_F = 57.757$

$M\_ED\_F := M\_ED\_0 + M\_ED\_1 \cdot \Delta_1 + M\_ED\_2 \cdot \Delta_2 \qquad M\_ED\_F = -11.121$


60.000
77.176
67.757
57.757
11.121
31.417
21.882
52.578
8.959


101.884
60.471
51.659
−10.000
−19.529
−28.341
−50.000
−49.529
−98.116


110.471 kN
151.413 kN
149.775 kN
28.341 kN
11.121 kNm